REDUCTION IN PIPE RESISTANCE IN THE FLOW OF A TWO-PHASE MEDIUM

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A discussion is presented of the Farrows–Lindquist effect — a sharp reduction in the resistance of a pipe to the flow of a viscous fluid containing solid matter.

The Farrows–Lindquist effect has been observed in the motion of blood through capillaries of a certain radius at Reynolds numbers considerably less than unity. The effect consists of the fact that the effective viscosity of whole blood obtained from Poiseuille's law turns out to be roughly half the viscosity of blood plasma [1]. It is well known that the main constituents of blood are red blood cells — biconcave disks — and that blood behaves as a Newtonian fluid in very narrow capillaries [1].

The authors of [2] solved a steady-state problem concerning the entrainment of an infinite series of solid spheres into a cylindrical pipe by a laminar flow of a viscous incompressible fluid. The Stokes approximation was examined. It was found that, under certain conditions, the resistance of the pipe in the entrainment of spherical particles by a viscous flow is about half the resistance of the pipe when Poiseuille flow is realized in it for a viscous fluid.

In [3], the Navier–Stokes equations were used to obtain a numerical solution to a similar problem. A study was made of the entrainment of two solid cylindrical particles by a flow of a viscous incompressible fluid in the Reynolds number range 0.001-10. It was assumed that the velocity distribution corresponded to Poiseuille flow at infinity. It was found that the pressure drop over a fairly long length of pipe containing the entrained particles is considerably (by 26%) less than the pressure drop for Poiseuille flow. This effect is seen not only at low Reynolds numbers, but also when the inertial terms of the Navier–Stokes equations are important. It should be noted that this effect occurs only in the case of particle entrainment, which is expressed by the requirement that the sum of the forces acting on the particle from the fluid be equal to zero.

Let us examine the flow pattern which develops in the entrainment of solid particles by a viscous flow (Fig. 1). The streamlines are expressed in the coordinate system in which the particles exist at rest. A zone of circulatory flow is typically formed between the entrainment particles. A similar flow was seen in [2] in the entrainment of spheres. The circulatory flow is formed because the entrainment velocity of the particles is somewhat greater than the mean velocity of the fluid in the pipe, and the particles and the fluid between them overtake the rest of the fluid. This flow plays the deciding role in the manifestation of the Farrows–Lindquist effect. It should be noted that the circulatory flow between the particles occurs with small velocity gradients, so that energy losses in it are negligible. As the calculations showed, the gain in the pressure drop which occurs with particle entrainment takes place in the immediate vicinity of the particle. In the case of Poiseuille flow, the pressure gradient acting on the ends of a hypothetical fluid cylinder is balanced by the viscous drag acting on the lateral surface of the same cylinder. In the flow with particle entrainment, the pressure gradient acting on these particles is balanced by the sum of the following forces: 1) friction acting on the solid particles; 2) friction acting on the fluid between the particles (this force is small, since the circulatory flow acts as a bearing and

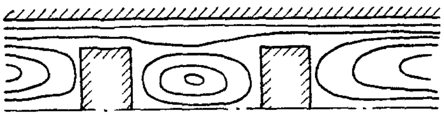

Fig. 1. Diagram of fluid flow with particles in a pipe.

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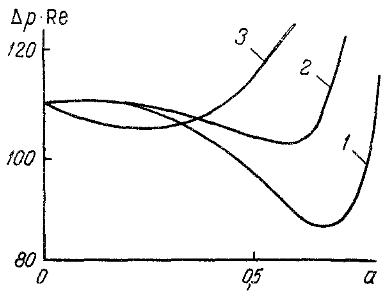

Fig. 2. Pressure gradient in the pipe in relation to the dimensions of the particles and the Reynolds number.

reduces friction); 3) pressure which develops in the region of circulatory flow between particles.

It follows from the calculations we performed that this sum, with particle entrainment, may be considerably less than the friction acting on the lateral surface of an equivalent fluid cylinder in the case of Poiseuille flow.

Figure 2 shows the pressure gradient multiplied by the Reynolds number over 3.4 pipe lengths in relation to the radius of the entrained cylindrical particles (here and below, all of the distances are referred to the pipe radius). Curve 1 corresponds to the entrainment of two particles of the length 0.125, with a distance between them equal to 0.75; curve 2 corresponds to the entrainment of two particles of the length 0.25, with a distance between them of 0.5; curve 3 corresponds to the entrainment of one particle of the length ℓ. The pressure gradient for Poiseuille flow on the graph for the same pipe length corresponds to a particle radius of zero. This gradient, multiplied by the Reynolds number, is equal to 108.8. Each curve has a pressure-gradient minimum. These minima are 26.5, 12.9, and 2.1% less than for Poiseuille flow on curves 1, 2, and 3, respectively. The volume of the entrained particles and the fluid between the particles is the same in all of these cases. This reduction in pressure drop cannot be explained by the fact that a substantial volume of fluid has been removed from the flow. Kinetic energy is not dissipated in these volumes, and they are the same for all three cases. It can be seen from the curves that the magnitude of the reduction in pipe resistance depends heavily on the length and radius of the entrained particles. It is evident that pipe resistance decreases even more with the entrainment of an infinite series of cylindrical particles of a certain radius.

Thus, the phenomenon of a reduction in pipe resistance with the entrainment of solid particles by a viscous fluid can be seen for steady laminar flow within a fairly broad range of Reynolds numbers, and it can be seen for a fairly broad range of particle shapes. When the shapes of the particles are dissimilar, the velocity distribution in the entrainment flow should be such that the velocity of entrainment of the particles is greater than the mean velocity of the fluid and circulatory flow develops between the particles.

NOTATION

p, pressure gradient; Re, Reynolds number; a, particle radius.

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